Self-alignment of Kinematic Couplings: Effects of Deformations

Francesco Patti
John Vogels
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A kinematic Coupling is an interface that can position parts with a repeatability up to sub-micron level: this can happen only if the coupling is self-aligning.

This study focusses on the self-aligning property of kinematic couplings, and how the deformations of the involved bodies can affect it.
Non conventional kinematic coupling:

In this case deformations and self-alignment are strongly connected
Introduction

What is the **self-aligning** property?

*When the body to be positioned is placed in an off-centered position, will it spontaneously move until it reaches the centered position?*

In general, the answer is yes only if the friction between the parts is smaller than a certain value, the so called *limiting coefficient of friction*: aim of this study is to evaluate such a value.
The system is not moving yet, but the situation is such that friction is at the maximum of its strength; any reduction of the friction coefficient would break the equilibrium. Such coefficient is the limiting coefficient of friction.
Strategy: Equilibrium – Incipient motion

Friction forces are needed to write the equilibrium equations

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All reactions will be lying on the friction cones:

\[ T \leq \mu N \]

\[ \mu = \tan \varphi \]

Friction cone
Contact points

2 contact points per each V-groove
Direction of friction forces

For each contact point, where is friction going to be oriented?

1. Classic approach: **against the motion.**
   *This assumption is true only if the coupling is actually moving.*

2. Our proposal: **anywhere.**
   *This means that the direction of friction forces cannot be found by studying the motion: extra equations are needed.*
1) Classic Approach

Knowing the incipient motion, we also know friction orientation. **Which motion should be considered?**
The most difficult path is among the 6 paths with 1 constraint removed.

Six 1 DoF aligning paths are investigated, by removing one by one all of the 6 contacts. This is a relatively simple problem (eigenvalue).
2) The New Approach

A way to determine the friction forces orientations is to treat the problem as hyperstatic (statically undetermined): the involved bodies are therefore considered deformable. This provides a number of extra equations, allowing to solve the problem.

The orientation of friction is determined by the way the system deforms.
2) The New Approach

Example

Expected displacement direction

Friction towards motion

Friction against motion
Worst Case Scenario

Looking for the orientation of friction forces that gives friction the maximum efficiency in preventing the aligning motion.

Five more parameters $\Omega_i$ are introduced: direction of friction forces.

Looking for worst case $\Omega_i$ such that $\mu$ is minimum.
Comparison between the two methods
Comparison between the two methods

![Graph showing comparison between two methods]

- Limiting Friction coefficient
- \( \beta \)
Example: asymmetric KC

Classic approach:
\[ \mu = 0.53 \]

Worst case:
\[ \mu = 0.19 \]
Conclusions

For standard kinematic couplings, symmetrical load, the self-aligning property can be evaluated without involving the study of deformations.

However, for cases that differ significantly from the previous ones, a more detailed study is required. A worst cases analysis is available, which helps to prevent underestimating the efficiency of friction.
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Using the usual approach to solve hyperstatic structure (1 extra compatibility equation is added), the solution is unique, and it is function of internal deformations of the system.

In this analysis motion and friction coefficients are not considered.

- The direction of friction in incipient motion is not affected by the possible motion.
- Not all friction reaction will lie on friction cones.
Equations

Direction of displacements

\[ d \delta_i = d \delta_0 + d \theta \times \vec{r}_i \quad i = 1, \ldots, 6 \text{ except } j \]

\[
\begin{cases}
    (d \delta_0 + d \theta \times \vec{r}_i) \cdot \vec{n}_i = 0 & i = 1, 2, \ldots, 6 \text{ except } j \\
    (d \delta_0 + d \theta \times \vec{r}_j) \cdot \vec{n}_j = -1
\end{cases}
\]

Eigensystem

\[
\begin{cases}
    c \vec{F}_e + \sum_{i \neq j} \left( N_i \vec{n}_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \right) = 0 \\
    \vec{r}_e \times c \vec{F}_e + \sum_{i \neq j} \vec{r}_i \times \left( N_i \vec{n}_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \right) = 0
\end{cases}
\]

Friction in any direction

\[
\begin{cases}
    \vec{F}_e + \sum_{i \neq j} \left( N_i \vec{n}_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \cos \Omega_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \times \vec{n}_i \sin \Omega_i \right) = 0 \\
    \vec{r}_e \times \vec{F}_e + \sum_{i \neq j} \vec{r}_i \times \left( N_i \vec{n}_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \cos \Omega_i - \mu N_i \frac{d \delta_i}{|d \delta_i|} \times \vec{n}_i \sin \Omega_i \right) = 0
\end{cases}
\]
Equations

Eigensystem in detail

\[ A = \begin{bmatrix}
  n_{1x} & \cdots & n_{5x} & F_{ex} \\
  n_{1y} & \cdots & n_{5y} & F_{ey} \\
  n_{1z} & \cdots & n_{5z} & F_{ez} \\
  (\bar{r}_1 \times \bar{n}_1)_x & \cdots & (\bar{r}_5 \times \bar{n}_5)_x & (\bar{r}_1 \times \bar{F}_e)_x \\
  (\bar{r}_1 \times \bar{n}_1)_y & \cdots & (\bar{r}_5 \times \bar{n}_5)_y & (\bar{r}_1 \times \bar{F}_e)_y \\
  (\bar{r}_1 \times \bar{n}_1)_z & \cdots & (\bar{r}_5 \times \bar{n}_5)_z & (\bar{r}_1 \times \bar{F}_e)_z 
\end{bmatrix} \]

\[ X = \begin{bmatrix}
  N_1 \\
  N_2 \\
  N_3 \\
  N_4 \\
  N_5 \\
  \mu \end{bmatrix} \]

\[ B = \begin{bmatrix}
  -\frac{d\delta_{1x}}{d\delta_i} & \cdots & -\frac{d\delta_{5x}}{d\delta_i} \\
  \frac{d\delta_{1y}}{d\delta_i} & \cdots & \frac{d\delta_{5y}}{d\delta_i} \\
  \frac{d\delta_{1z}}{d\delta_i} & \cdots & \frac{d\delta_{5z}}{d\delta_i} \\
  -\frac{d\delta_{1x}}{d\delta_i} & \cdots & -\frac{d\delta_{5x}}{d\delta_i} \\
  -\frac{d\delta_{1y}}{d\delta_i} & \cdots & -\frac{d\delta_{5y}}{d\delta_i} \\
  -\frac{d\delta_{1z}}{d\delta_i} & \cdots & -\frac{d\delta_{5z}}{d\delta_i} \\
\end{bmatrix} \]

\[ AX + \mu BX = 0 \]
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